A STATISTICAL ANALYSIS OF MELODY LENGTHS IN A NORTH INDIAN RAGA

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Abstract
The length of a melody simply refers to the number of notes in it. Melody lengths also provide interesting data sets that merit statistical attention individually, apart from their role in calculating melody significance. However, their individual study has been a neglected aspect in music analysis. This paper makes amends with a case study in a north Indian *raga*. A 0-2 truncated Poisson model seems to characterize the melody lengths.

***Keywords*:** raga, melody, melody length, truncated Poisson model, statistical analysis.

INTRODUCTION

In music, the term **melody** may be mathematically defined as a sequence of notes "complete" in some sense as determined by music theory, taken from a musical piece.[1] A melody need not be a complete musical sentence. It suffices if it is a complete musical phrase. A **Segment** is a sequence of notes which is a subset of melody but is itself incomplete. For example, {Sa, Sa, Re, Re, Ga, Ga, Ma, Ma, Pa} is a melody and {Sa, Sa, Re Re} is its segment in raga *Kafi*. **Length** of a melody or its segment refers to the number of notes in it. **Significance** of a melody or its segment (in monophonic music such as Indian classical music) is defined as the product of the length of the melody and the number of times it occurs in the musical piece. *Thus, both frequency and length are important factors to assess the significance of a melody or its segment*.[2]

Melody lengths also provide interesting data sets that merit statistical attention individually, apart from the role in calculating melody significance, and yet their individual study has been a neglected aspect in music analysis. This paper makes amends with a case study in a north Indian *raga*, namely, *Malkauns*. A raga is a melodic structure with fixed notes and a set of rules characterizing a certain mood conveyed by performance.[3]

Appendix (Table 5) gives the sequence of notes of raga *Malkauns* taken from a standard text.[4] Table 1 gives the formation of melody groups as judged by the first author by rendering each melody group in a

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[1] Soubhik Chakraborty, Kolla Krishnapriya, Loveleen, Shivee Chauhan and Sandeep Singh Solanki, "Analyzing the melodic structure of a North Indian raga: A statistical approach", *Electronic Musicological Review*, Vol. XII, April 2009.

[2] For a more technical definition of significance of melody in polyphonic music, see Kamil Adiloglu, Thomas Noll & Klaus Obermayer, "A Paradigmatic Approach to Extract the Melodic Structure of a Musical Piece", *Journal of New Music Research*, Vol. 35, Issue 3, 2006, p. 221–236.

[3] Soubhik Chakraborty *et al., art. cit.*

[4] Debabrata Dutta, *Sangeet Tattwa* (Pratham Khanda), Brati Prakashani, 5th ed., 2006 (in Bengali).

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harmonium and assessing completeness. Although intuitive, it should be understood that *behind this aesthetic intuition it is Indian music theory that works*.[5]

Table 2 gives the frequency distribution of melody lengths, built with the help of Table 1. Fig. 1 gives a histogram of the frequency distribution. This is followed some descriptive statistical features of the melody lengths as provided by MINITAB 15.0 statistical package.

Table 1

Melody groups and their lengths

The melody groups are as under:

Melody group No.	Length (no. of notes)	Frequency	Significance = length x frequency
G1: 1-7	07	1	7
G2: 8-16	09	1	9
G3: 17-25	09	1	9
G4: 26-34	09	1	9
G5: 35-42	08	1	8
G6: 43-48	06	3	18
G7: 49-56	08	1	8
G8: 57-67	11	2	22
G9: 68-77	10	1	10
G10: 78-88	11	1	11
G11: 89-94	06	1	06
G12: 95-103	09	1	09
G13: 104-109	06	2	12
G14: 110-128	19	1	19
G15: 129-134	06	2	12
G16: 135-139	05	7	35
G17: 140-147	08	1	8
G18: 148-159	12	1	12
G19: 160-171	12	1	12
G20: 172-180	09	2	18
G21: 181-190	10	2	20
G22: 191-196	06	2	12
G23: 197-208	12	1	12
G24: 209-217	09	1	9
G25: 218-226	09	2	18
G26: 227-238	12	1	12
G27: 239-254	16	1	16
G28: 255-262	08	1	8

Table 2

Frequency distribution of melody groups

Melody length	frequency	relative frequency
5	01	1/28
6	05	5/28
7	01	1/28
8	04	4/28
9	07	7/28
10	02	2/28
11	02	2/28
12	04	4/28
13	00	00
14	00	00
15	00	00
16	01	1/28
17	00	00
18	00	00
19	01	1/28
TOTAL	28	1

[5] For a sound literature on Hindustani classical music, see N. A. Jairazbhoy, *The rags of North India: Their Structure and Evolution*, Faber and Faber, London, 1971.

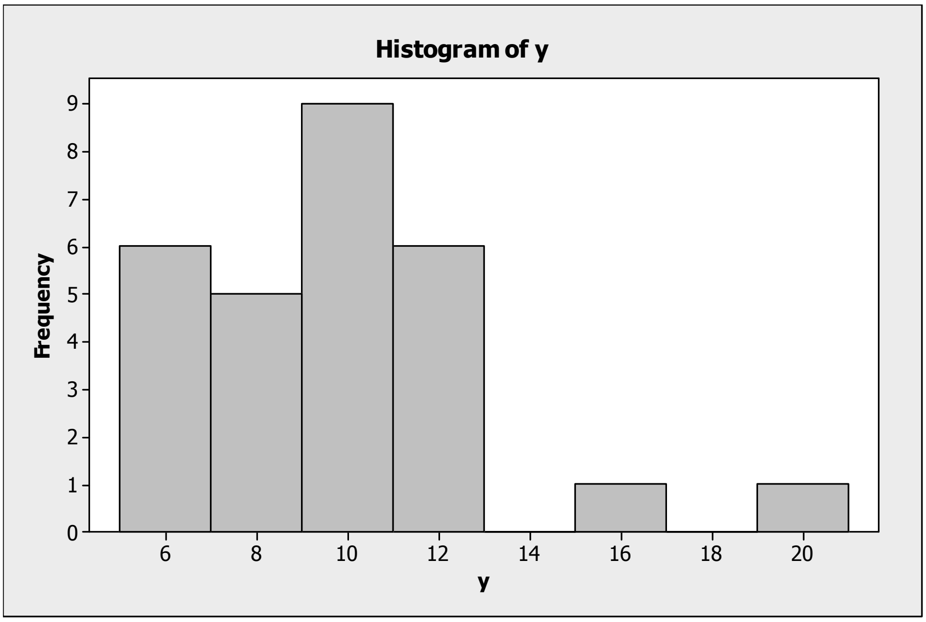

Fig. 1 – Histogram showing the distribution of melody lengths.

Descriptive Statistics: y (melody length)

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
y	28	0	9.357	0.587	3.106	5.000	7.250	9.000	11.000

Variable	Maximum	IQR	Mode	N for Mode	Skewness	Kurtosis
y	19.000	3.750	9	7	1.30	2.53

Remark: "N for mode is 7" means the mode (9) is repeated 7 times in the distribution. Variance = Square of standard deviation = 9.647 (approx.)

Fig. 2 gives a time series plot of the 28 melody lengths (y) sequentially against instances 1, 2…28 (called index in the figure).

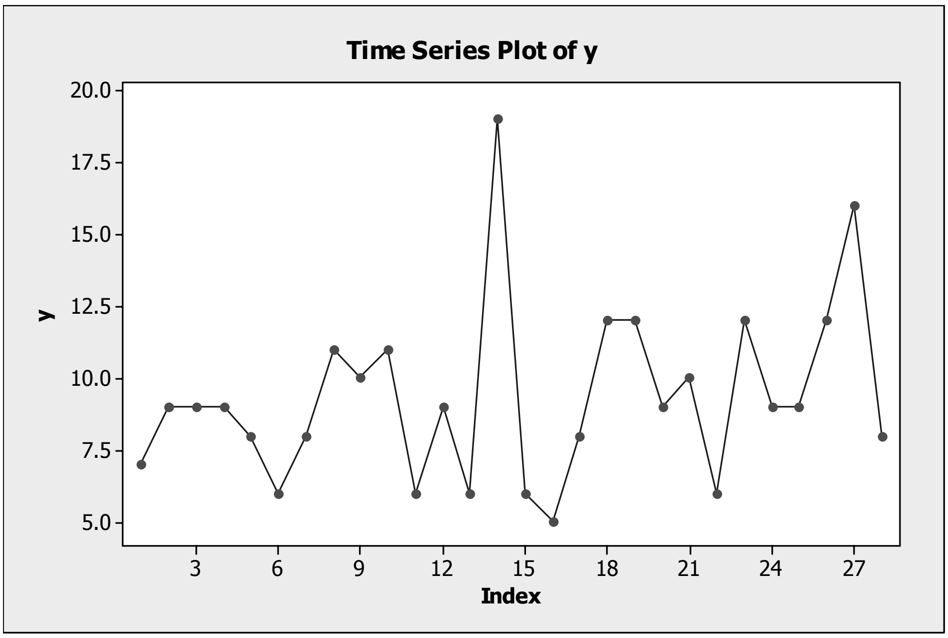

Fig. 2 – Time series plot of the 28 melody lengths (y) sequentially against instances.

DISCUSSION

The mean melody length for the present data set in *raga Malkauns* is 9.357 with a standard deviation of 3.106. The melody lengths are in the range [5, 19] for this sample. Mean > median = mode implying positive skewness. The measure of kurtosis is less than 3 implying the frequency curve is platykurtic. In what follows, we shall fit a truncated Poisson model to characterize the distribution of the melody lengths.

Poisson distribution (definition): A discrete random variable is said to have a Poisson distribution with parameter λ if its probability mass function (pmf) is given by

$P(X=x) = (e^{-\lambda} \lambda^{x})/x!$; x=0, 1, 2, 3….

= 0, otherwise

Writing P(x) as the pmf of Poisson distribution for brevity,

We must have P(0)+P(1)+P(2)+ P(3)+……=1 (probability over the entire range)

Or, P(3) + P(4) +……=1-P(0)-P(1)-P(2)

Hence the pmf of a truncated Poisson distribution, with truncation points at 0, 1 and 2 is given by (1/c)P(x) where c=1-P(0)-P(1)-P(2)

The logic behind fitting a Poisson model is that mean and variance are nearly equal. The logic behind choosing a 0-2 truncated Poisson model is that a melody must at least have a length of 3 to be a "complete" musical phrase if not a complete musical sentence(otherwise it is a segment). A truncated Poisson model is also a Poisson model that cannot take the truncated values. The reader may have observed that this amounts to adjusting only a constant factor (1/c); this does not change the nature of the distribution. We will of course verify the goodness of fit.

Taking mean of Poisson distribution as 9.357, we compute c=1.004697

The rest of the calculations are given in Table 3 below:

Table 3

Fitting a 0-2 truncated Poisson model to melody lengths

P(x)	(1/c)*P(x)	Expected frequency=28*(1/c)*P(x)
P[3]=0.011791	0.011847	0.33171
P[4]=0.027583	0.027713	0.775952
P[5]=0.051619	0.051861	1.452116
P[6]=0.0805	0.080878	2.264576
P[7]=0.107605	0.10811	3.027091
P[8]=0.125857	0.126449	3.540561
P[9]=0.13085	0.131464	3.681003
P[10]=0.122436	0.123011	3.444315
P[11]=0.104149	0.104638	2.92986
P[12]=0.08121	0.081591	2.284558
P[13]=0.058452	0.058727	1.644355
P[14]=0.039067	0.039251	1.099016
P[15]=0.02437	0.024485	0.705648 (adjusted to sum to 28)
P[16]=0.014252	0.014319	0.400928
P[17]=0.007844	0.007881	0.220675
P[18]=0.004078	0.004097	0.114714
P[19]=0.002008	0.002018	0.056494
P[20]=0.00094	0.000944	0.026431

Writing O for observed frequency and E for expected frequency, ensuring the E values are not less than 5 for each class and pooling classes from previous table, we have the next table (Table 4).

Chi-Square= $\Sigma(O-E)^2/E$ summed over all the three classes = 1.225 with 3-2=1 degree of freedom (one degree of freedom is lost because of restricting sum of O values to equal sum of E values; one degree of freedom is lost as the parameter λ was estimated from the musical data). Classes are pooled to make each cell frequency at least five as otherwise Chi-Square, which is a continuous distribution, cannot maintain its character of continuity. Table value of Chi-Square at 1 degree of freedom and 5% level of significance is 3.841. For further literature in probability distributions, Ross may be consulted.[6] For the descriptive statistics, we refer to Walpole, Myers, Myers and Ye.[7] The calculated value of Chi-square being less than the tabulated value, the fit is good at 5% level of significance. Hence we will accept the 0-2 Truncated Poisson model.

Table 4

Chi-Square Goodness of fit for 0-2 truncated Poisson model

Classes	O	E
1. (melody length 3-6)	6	4.84
2. (melody length 7-9)	12	10.26
3. (melody length 10-above)	10	12.9
Total	**28**	**28**

CONCLUDING REMARKS

The present paper clearly shows why melody lengths are important in their own right in music analysis and the role of statistics in exploring the same cannot be thrown away as are evident through the descriptive features and modeling. True, our 0-2 truncated Poisson model is only a subjective working model. *All statistical models are subjective, but they arise from data that is objective – or nearly so – and often, at their center, stand some very elegant mathematical theorems – of course, free from bias*.[8] One of the major strengths of statistics lies in modeling, given that the true model is often unknown and complex and we can always verify the goodness of fit of a working model. So it all depends on what the statistician expects from the fitted model and how far the model meets the requirement. Future work involves studying melody lengths of other ragas as well as comparing melody lengths of different ragas or of the same raga performed by various artists. The latter part of course will involve the additional task of note extraction with onset (point of time of arrival of the note) involving acoustical signal processing of the corresponding audio samples. From the calculations of the significance of each melody in Table 1, we find that the melody group 16 or more precisely the melody {M g M d M} has the highest significance: 35.

[6] Sheldon M. Ross, *A First Course in Probability*, 7^{th} ed., Pearson Edu., 2006.
[7] R. E. Walpole, R. H. Myers, S. L. Myers and K.Ye, *Probability and Statistics for Engineers and Scientists*, Pearson Edu., 2007.
[8] Ben Klemens, *Modeling with data: tools and techniques for scientific computing*, Princeton University Press, Princeton, NJ, 2008.

APPENDIX

Table 5

Malkauns note sequence[9]

(Instance) Serial No.	Note	Pitch									
1	S	0	65	M	5	131	S	0	197	d	8
2	M	5	66	g	3	132	S	0	198	n	10
3	g	3	67	S	0	133	*n*	-2	199	M	5
4	M	5	68	*D*	-4	134	S	0	200	g	3
5	M	5	69	*n*	-2	135	M	5	201	g	3
6	g	3	70	S	0	136	g	3	202	M	5
7	S	0	71	S	0	137	M	5	203	d	8
8	*n*	-2	72	S	0	138	d	8	204	M	5
9	S	0	73	*n*	-2	139	M	5	205	g	3
10	*d*	-4	74	S	0	140	M	5	206	M	5
11	*M*	-7	75	*d*	-4	141	g	3	207	g	3
12	*n*	-2	76	*n*	-2	142	M	5	208	S	0
13	*d*	-4	77	S	0	143	d	8	209	*d*	-4
14	S	0	78	g	3	144	M	5	210	*n*	-2
15	*n*	-2	79	S	0	145	n	10	211	S	0
16	S	0	80	*d*	-4	146	d	8	212	M	5
17	*n*	-2	81	*n*	-2	147	M	5	213	g	3
18	S	0	82	S	0	148	g	3	214	M	5
19	g	3	83	M	5	149	M	5	215	d	8
20	M	5	84	M	5	150	g	3	216	n	10
21	d	8	85	g	3	151	n	10	217	**S**	12
22	M	5	86	M	5	152	d	8	218	d	8
23	n	10	87	g	3	153	M	10	219	n	10
24	d	8	88	S	0	154	g	3	220	**S**	12
25	**S**	12	89	*n*	-2	155	M	5	221	d	8
26	**S**	12	90	*d*	-4	156	d	8	222	n	10
27	n	10	91	*n*	-2	157	M	5	223	d	8
28	d	8	92	S	0	158	g	3	224	**S**	12
29	M	5	93	*n*	-2	159	S	0	225	n	10
30	g	3	94	S	0	160	*n*	-2	226	**S**	12
31	M	5	95	M	5	161	S	0	227	d	8
32	g	3	96	g	3	162	*d*	-4	228	n	10
33	*n*	-2	97	M	5	163	*n*	-2	229	d	8
34	S	0	98	M	5	164	S	0	230	**g**	15
35	S	0	99	g	3	165	M	5	231	d	8
36	M	5	100	M	5	166	g	3	232	n	10
37	g	3	101	g	3	167	M	5	233	d	8
38	M	5	102	*n*	-2	168	n	10	234	**M**	17
39	d	8	103	S	0	169	d	8	235	**g**	15
40	n	10	104	*d*	-4	170	n	10	236	**M**	17
41	d	8	105	*n*	-2	171	**S**	12	237	**g**	15
42	M	5	106	S	0	172	d	8	238	**S**	12
43	M	5	107	S	0	173	n	10	239	d	8
44	g	3	108	*n*	-2	174	**S**	12	240	n	10
45	M	5	109	S	0	175	d	8	241	**S**	12
46	d	8	110	M	5	176	n	10	242	n	10
47	n	10	111	g	3	177	d	8	243	d	8
48	**S**	12	112	M	5	178	**S**	12	244	n	10
49	n	10	113	d	8	179	n	10	245	d	8
50	**S**	12	114	M	5	180	**S**	12	246	M	5
51	**g**	15	115	M	5	181	d	8	247	g	3
52	**S**	12	116	g	3	182	n	10	248	M	5
53	n	10	117	M	5	183	d	8	249	d	8
54	**S**	12	118	d	8	184	M	5	250	M	5

[9] Debabrata Dutta, *op. cit*.

55	n	10	119	M	5	185	M	5	251	g	3
56	d	8	120	g	3	186	g	3	252	M	5
57	n	10	121	M	5	187	M	5	253	g	3
58	d	8	122	d	8	188	d	8	254	S	0
59	M	5	123	M	5	189	n	10	255	*n*	-2
60	g	3	124	g	3	190	**S**	12	256	S	0
61	M	5	125	M	5	191	n	10	257	*d*	-4
62	d	8	126	g	3	192	**S**	12	258	*n*	-2
63	M	5	127	*n*	-2	193	**g**	15	259	S	0
64	g	3	128	S	0	194	**S**	12	260	M	5
			129	*d*	-4	195	n	10	261	g	3
			130	*n*	-2	196	**S**	12	262	S	0

Numbers representing pitch of notes:[10]

C	Db	D	Eb	E	F	F#	G	Ab	A	Bb	B	
S	*r*	*R*	*g*	*G*	*M*	*m*	*P*	*d*	*D*	*n*	*N*	**(lower octave)**
-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	
S	r	R	g	G	M	m	P	d	D	n	N	**(middle octave)**
0	1	2	3	4	5	6	7	8	9	10	11	
S	**r**	**R**	**g**	**G**	**M**	**m**	**P**	**d**	**D**	**n**	**N**	**(higher octave)**
12	13	14	15	16	17	18	19	20	21	22	23	

Abbreviations: The letters S, R, G, M, P, D and N stand for Sa, *Sudh* Re, *Sudh* Ga, *Sudh* Ma, Pa, *Sudh* Dha and *Sudh* Ni respectively. The letters r, g, m, d, n represent *Komal* Re, *Komal* Ga, *Tibra* Ma, *Komal* Dha and *Komal* Ni respectively. Normal type indicates the note belongs to middle octave; italics implies that the note belongs to the octave just lower than the middle octave while a bold type indicates it belongs to the octave just higher than the middle octave. Sa, the tonic in Indian music, is taken at C. Corresponding Western notation is also provided. The terms "*Sudh*", "*Komal*" and "*Tibra*" imply, respectively, natural, flat and sharp.

Basic features of raga *Malkauns*:[11]

Thaat (raga-group according to the notes used): *Bhairavi* (some experts say *Asavari*[12])

Aroh (ascent): S g M d n **S** (**S** is of higher octave); *Awaroh* (descent): **S** n d M, g M g S

Jati: *Aurabh-Aurabh* (five distinct notes used in ascent; five again in descent)

Vadi Swar (most important note): M; *Samvadi Swar* (second most important note): S

Prakriti (nature): Restful and serious; *Pakad* (catch): M g, M d n d, M g S

Nyas swar (Stay notes): g M and d; Time of rendition: 3rd phase of night (12 PM - 3 AM)

Rajan Parrikar, an acknowledged expert in Indian classical music, adds:

"In Pandit Bhatkhande's taxonomic scheme the rAga is placed under the Bhairavi thAT. Recall that certain gestures in Raga Bhairavi take after Malkauns. The signal characteristic of Malkauns concerns its nyAsa locations: each one of its swaras is considered apposite for nyAsa. No other rAga has this attribute. The implication being, you cannot go wrong in Malkauns. With five locations available for nyAsa, the vistAr area extends far and wide, overcoming the limitations of a restricted tonal space inherent to pentatonic rAgas."

The *chalan* or melodic movement is beautifully described by Parrikar as follows:

[10] See Chakraborty *et al., art. cit.*
[11] See Debabrata Dutta, *op. cit.*
[12] *Ibid.*

"The key lakshaNAs of Malkauns are now encapsulated:

S, d' n' S, n' g->S, S g M g->S
The sui generis meeND **g->S** serves as a vital constituent of Raganga Kauns. The ucchAraNa is crucial: **g** is rendered deergha before initiation of the meeND. This precipitates the shAnta-gambheera rasa characteristic of Kauns. The langhan (skipping) of **n** in the declining prayoga from **S** to **d'** (or from **S"** to **d)** is another point of note.

d' n' S M, M d M, S g M g->S
Although all the swaras are nyAsa-worthy, the madhyam is considered *primus inter pares*, the centre of melodic gravity, as it were. Notice the leap from **S** to **M,** a Malkaunsian trait.

g M d n, M d n S" d n d->M
Here we have uttarAnga modus operandi. The **d->M** meeND is analogous to the poorvAnga **g->S** and serves as another artifact of the Raganga Kauns kernel; **d** is rendered deergha before the slide to **M,** again reinforcing the shAnta-gambheera effect.

S", d n S" g", g" M" g", n g"->S
Attention is drawn to the **n** to **g** coupling, a key lakshaNA of Raganga Kauns.

Obiter dicta:
(a) In the Malkauns progression there is samvAd (consonance) between the **S-M, g-d** and **M-n** pairs.
(b) It is observed that some vocalists occasionally admit a higher value of **n** that falls within the penumbra of **N,** especially in the Arohi prayogas. This practice appears to be inadvertent given the inconsistency and irregularity with which it occurs".[**]

***Acknowledgements**: The first author thanks Rajan Parrikar for the permission to quote from his musical blog (http://www.parrikar.org/raga-central/malkauns/).*

[**] In Parrikar's notation the single quote after a note indicates the note belongs to lower octave and a double quote indicates the note being in higher octave. Absence of a quote represents the middle octave.